The Demarcation of Science

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Demarcation

Demarcation means the act of establishing the boundary or limits of something.

Overview

Over the past fifty years Bayesian inference has become the dominant theory of scientific method. This presentation argues not only that the Bayesian paradigm represents a credible scientific method, but that science essentially *is* applied Bayesian analysis. More precisely, science is intelligent applied subjective Bayesian analysis constrained by exchangeability, the Reflection Principle and the Principal Principle.

To finish, the presentation shows that induction and abduction are in essence scientific, whilst Popper's falsification is not.

Science

What is science? Dictionary definitions inform us that *science* is the systematic study of the universe—through observation and experiment—in the pursuit of knowledge that allows us to *generalize*.

Here we are interested in a *normative* approach to science, we aim to describe an idealized agent.

Subjective Assumptions

Hume (1739–40) pointed out that 'even after the observation of the frequent or constant conjunction of objects, we have no reason to draw any inference concerning any object beyond those of which we have had experience'. More recently, and with increasing rigour, Mitchell (1980), Schaffer (1994) and Wolpert (1996) showed that bias-free learning is futile. The important point is that one can never generalize beyond one's data without making subjective assumptions, in other words, science involves a degree of uncertainty.

Science is. . . whatever?

So if science is about generalizing, but generalizing from first principles is impossible, where does that leave science? Science clearly involves making assumptions, but that does not mean that 'anything goes', because we can insist upon intimating one's degree of uncertainty and self-consistent reasoning.

Dutch Book

A *Dutch book* is a gambling term for a set of odds and bets that guarantees a profit, regardless of the outcome of the gamble. For example, consider a bookmaker who quotes the following odds (implied by his degrees of belief) for the result of the toss of a coin.

	Degree of belief	Odds against	Bet	Payout
Heads	0.5	1/1	£1.25	£2.50
Tails	0.4	3/2	£1.00	£2.50
Heads $\vee$ Tails	0.9		£2.25	£2.50

A punter can bet on both outcomes, as above, and his profit will always be £2.50 - £2.25 = £0.25, so he has made a Dutch book against the bookie.

Probability

At the very least, one who practices self-consistent reasoning should not be susceptible to having a Dutch book made against them. If an individual is not susceptible to a Dutch book, their previsions are said to be *coherent*. A set of betting quotients is coherent if (Ramsey 1926; de Finetti 1937; Shimony 1955) and only if (Kemeny 1955; Lehman 1955) they satisfy the axioms of probability.

Thus far, we have shown that the scientific method necessitates making subjective assumptions and following the rules of probability.

Subjective Bayesian Analysis

Bayes' theorem is merely the calculus for updating a probability in the light of new evidence, so the validity of the formula itself is not controversial, but it does presuppose the applicability of probability. By definition, an individual is a Bayesian to the extent that they are willing to put a probability on a hypothesis. Incorporate the admission that subjective assumptions are necessary, and science becomes subjective Bayesian analysis.

Self-Consistent Reasoning

We have argued that self-consistent reasoning involves avoiding a Dutch book, but we can go further. Self-consistent reasoning also involves treating cases of symmetry as symmetrical, being self-consistent across time and respecting truly random processes.

Exchangeability

Instances of symmetry (of events or permutations of events) should be treated symmetrically, and such judgement is known as *exchangeability* (de Finetti 1937). For example, in the absence of any other information, the probability assigned to throwing a six with a fair die should be $\frac{1}{6}$.

Reflection Principle

To ensure that one is self-consistent across time, one's current beliefs must equal one's current beliefs about one's future beliefs, which is known as the *Reflection Principle* (van Fraassen 1984, 1995). For any hypothesis, H, time $t \geq 0$ and $0 \leq x \leq 1$

$$P(H|P_t(H) = x) = x.$$

For example, the UEFA Euro 2012 final tournament will be hosted by Poland and Ukraine between 8th June and 1st July 2012.
$P_{(\text{today})}(\text{England win Euro 2012}) = \frac{1}{12}$
$P_{(\text{7th June 2012})}(\text{England win Euro 2012}) = \frac{1}{12}$

Principal Principle

The *Principal Principle* (Lewis 1980) provides a mechanism for converting an objective probability into a subjective probability. So when truly random processes are involved (i.e. only phenomena which are aspects of quantum mechanics), the Principal Principle should be observed. Formally,

$$P(H|P_{obj}(H) = x) = x.$$

For example, your degree of belief that a given radioactive atom will decay within a certain period of time must equal the objective probability that it will do so.

Intelligence

Despite, and in addition to, following the above principles designed to enforce self-consistent reasoning, we are still left with the problem of how best to make subjective assumptions. *Intelligence* is the ability of an individual to perform a novel cognitive task (Carroll 1993), whilst, similarly, the subjective element of science entails assigning a prior probability to a novel hypothesis. Science requires intelligence, which is borne out historically.

Conclusion

To conclude, science is intelligent applied subjective Bayesian analysis constrained by exchangeability, the Reflection Principle and the Principal Principle.

Scientific Methods

Finally, let us consider the implications of a Bayesian interpretation of science. When comparing hypotheses we can use the following version of Bayes' theorem, where D is data

$$P(H|D) \propto P(H)P(D|H).$$

Utilizing the above, how do the traditional scientific methods fair?

- Enumerative *induction* seeks to maximize $P(H|D)$ directly.
- Popper's *falsification* generalizes to maximizing $P(D|H)$.
- The aim of *abduction* (also known as *inference to the best explanation*) is to maximize $P(H)P(D|H)$ where D consists of facts.

Induction and abduction are in essence scientific, whilst Popper's falsification is an incomplete notion of science.

Popper's Falsification

- Requires an infinite number of hypotheses
- Is not robust
- Fails with existential statements
- Fails with probabilistic statements
- Fails in practice anyway due to the necessity of auxiliary assumptions

How has Popper's falsification performed historically? Newton's gravitational theory, Bohr's theory of the atom, kinetic theory, the Copernican Revolution and the theory of evolution were all falsified, despite being excellent examples of science. A corollary of the demarcation of science outlined here, that Popper's falsification is inadequate, is borne out in both theory and practice.

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